Refreshing the Math Shelves

Laura Opfer, M.Ed. and Adam Diamond, M.Ed.

Covering State or Common Core Standards

Montessori lessons and materials do a fabulous job of covering the majority of the lower elementary (as well as many of the upper elementary) learning standards that we have reviewed from the state of Ohio, which were originally derived from the Common Core Standards, then revised in 2017. However, there are some notable concepts that might be missing from your albums. Here are a few lesson plans that may be useful in utilizing our Montessori math materials to illuminate additional concepts:

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Comparing Numbers (Inequalities)

Domain: Number and Operations in Base Ten/Cluster: Understand Place Value

First Grade Standard (1.NBT.3)- Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, <, and =.

Second Grade Standard (2.NBT.4)- Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, <, and = symbols to record the results of comparisons.

Age: 6-9 years

Materials: Colored Bead Box, Golden Bead Material, 2 sets of small decimal cards, symbols box (wood, plastic or laminated paper) that includes >, < and = signs

First Presentation:

1.) Use the colored bead bars to form a two-digit number (e.g., 16). Have the child say the number and use the decimal cards to form it, placing the numerals below the beads.

2.) Form a second, different two-digit number with colored bead bars (e.g., 13). Again, have the child say the number and form the numerals with the cards.

3.) Ask the child which of the two numbers is larger. Give the child an opportunity to explain how they figured this out.

4.) Introduce the > and < symbols, as well as reviewing the = symbol. Using the symbols cards, place the appropriate symbol between the numbers formed with beads and/or the decimal cards.

5.) Model how to read the inequality number sentence ("Sixteen is greater than thirteen") and ask the child to do the same for each comparison.

5.) Repeat with a variety of two-digit numbers, establishing the idea that the child should look to the higher place value (in this case, the tens) first to determine which is larger, before counting the value of the ones place, if the tens are the same.

6.) The child can draw the beads with the correct symbol between and write the numbers below or simply write the numbers and symbols.

Second Presentation:
Repeat the same steps above with three-digit numbers, using the Golden Bead Material to form the numbers.

Extensions: Create addition, subtraction or multiplication equations on each side to be solved before the comparison can be made. You can also use the metal or plastic fraction pieces to create fraction inequalities for higher level students.

Comparing Fractions (Inequalities)

Domain: Numbers and Operations – Fractions/Cluster: Develop understanding of fractions as numbers

Third Grade Standard (3.NF.3.d) - Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Fourth Grade Standard (4.NF.2) - Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Age: 8-10 years

Materials: Box of metal/plastic fraction pieces, symbols box (wood, plastic or laminated paper) that includes >, < and = signs

Presentation:
1.) Place a one-half (½) piece from the box on the rug. Have the child say the fraction, pointing out the numerals printed on the piece.

2.) Place a different fraction piece (e.g., ⅓) to the right of the first piece, again having the child say the fraction name.

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3.) Ask the child which of the two fractions is larger. Give the child an opportunity to explain how they figured out, allowing (or demonstrating) superimposing of the fraction pieces if necessary.

4.) Review the >, <, and = symbols. Using the symbols cards, place the appropriate symbol between the fraction pieces.

5.) Model how to read the inequality number sentence ("One-half is greater than one-third") and ask the child to do the same for each comparison.

5.) Repeat with a variety of fractions. The child can record the information by tracing and coloring the pieces or simply writing the numerals. The child should practice until they identify the pattern that the *smaller* the denominator, the *larger* the fraction is.

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Money

Domain: Measurement and Data/Cluster: Work with time and money

First Grade Standard (1.MD.3)- Identify pennies and dimes by name and value.

Second Grade Standard (2.MD.8) - Solve problems with money. a.Identify nickels and quarters by name and value. b.Find the value of a collection of quarters, dimes, nickels, and pennies.

Money: Counting on Coins with Teens and Tens Boards

Age: 5-7 years

Prior Learning: Coin identification by name and value; Teens and Tens Boards

Materials: Teens Board, Tens Boards, coin value cards (from 11¢ to 19¢) a coin box with 100 pennies, at least 20 dimes, 20 nickels, and 8 quarters (you may also include half dollar, dollar coins, and paper bills)
For the first presentation of the Teen Board lesson you need 9 dimes and 45 pennies.

Presentation:

1. Say, "Today we are going to do something new with materials we have used before. We are going to learn to count money with the coins we have learned using the Teen Board to help us."

2. Review with the students and have them lay out their prior work making teens numbers with the boards and beads.

3. Say, "Our first number is 11. You put out 1 ten bar and 1 unit bead. Which coin have we learned that is worth the same as our ten bar- which coin is worth ten cents?" Child should answer that the dime is worth ten cents. Have the child place the dime to the right of the board next to the number 11. Say, "We also have a unit bead to represent the one unit in 11. Which coin is worth 1 cent?" The child should answer that a penny is worth one cent. Have the child place the penny to the right of the dime.

4. Show them the card that says 11¢ and say, "This is one way to write eleven cents." Have the child place the card to the right of the coins.

5. Continue in this manner with the values 12-19, laying the dimes and pennies horizontally to the right of the numeral cards and the coin value cards to the right of the coins.

Variations/Extensions: After using pennies successfully with the Teens Board, students can be taught to exchange a nickel for five pennies once you reach fifteen cents.

Students can use the Tens Boards in the same manner to lay out a variety of tens quantities up to 99 cents with dimes, nickels, and pennies.

Money

Domain: Measurement and Data/Cluster: Work with time and money

First Grade Standard (1.MD.3)- Identify pennies and dimes by name and value.

Second Grade Standard (2.MD.8) - Solve problems with money. a.Identify nickels and quarters by name and value. b.Find the value of a collection of quarters, dimes, nickels, and pennies.

Money Equivalence: Skip Counting Nickels and Dimes with the Short Bead Chains

Age: 5-7 years

Prior Learning: Coin identification (name and value) and dollar bill; Short Bead Chains

Materials: Short Five and Ten Chains; a coin box with 100 pennies, at least 20 dimes, 20 nickels, and 8 quarters (you may also include half dollar, dollar coins, and paper bills)
You will need 5 nickels for the first presentation and 10 dimes, plus 1 dollar bill (dollar coin) for the second presentation.

Presentation 1:

1. Say, "Today we are going to do something new with materials we have used before. We are going to learn to skip count with the coins we have learned using the Short Five Bead Chain to help us."

2. Review with the students and have them lay out their prior work, placing the number tags below along the Short Five Chain and skip counting by 5 to 25.

3. Say, "We have learned a coin that is worth five cents. Which coin is that?" Child should answer the nickel. Ask the child to place a nickel above the 5th bead at the end of the first bar (the tag for 5 should be below that last bead). Say, "One nickel is worth five cents."

4. Say, "When we skip count by 5, what is the number at the end of the next 5 bar?" Child should answer 10. "Say, please place a second nickel above the chain for 10. How much are two nickels worth? Child should answer ten cents.

5. Continue in this manner until you reach the fifth nickel. Ask, "How much are five nickels worth?" Child should answer twenty-five cents.

6. "Yes, five nickels are worth twenty-five cents. Can you remember another coin that is also worth twenty-five cents?" Child should answer "a quarter." Ask the child to place a quarter at the end of the chain.

7. You can skip count by fives to twenty-five cents together again and reiterate that five nickels is worth twenty-five cents and one quarter is also worth twenty-five cents.

Presentation 2:

1. Say, "Today we are going to do something new with materials we have used before. We are going to learn to skip count with the coins we have learned using the Short Ten Bead Chain (or 100 Chain) to help us."

2. Review with the students and have them lay out their prior work, placing the number tags below along the Short Ten Chain and skip counting by 10 to 100.

3. Say, "We have learned a coin that is worth ten cents. Which coin is that?" Child should answer the dime. Ask the child to place a dime above the 10th bead at the end of the first bar (the tag for 10 should be below that last bead). Say, "One dime is worth ten cents."

4. Say, "When we skip count by 10, what is the number at the end of the next 10 bar?" Child should answer 20. "Say, please place a second dime above the chain for 20. How much are two dimes worth? Child should answer "twenty cents."

5. Continue in this manner until you reach the tenth dime. Ask, "How much are ten dimes worth?" Child should answer "100 cents."

6. "Yes, ten dimes are worth 100 cents. Can you remember a different type of money that is also worth 100 cents?" Child should answer "a one dollar bill." Ask the child to place a dollar bill at the end of the chain.

7. You can skip count by tens to 100 cents together again and reiterate that ten dimes is worth 100 cents and a one-dollar bill is also worth 100 cents.

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Rounding with Whole Numbers

Domain: Number and Operations in Base Ten/Cluster: Use place value understanding and properties of operations to perform multi-digit arithmetic

Third Grade Standard (3.NBT.1)- Use place value understanding to round whole numbers to the nearest 10 or 100

*These rounding lessons are adapted from the Xavier University Montessori Teacher Education Program Mathematics for Elementary I Album, 2005

Rounding to the Nearest Ten*

Age: 7-8 years

Materials: short 10 chain (100) and number arrows, 100 square, counter, an arrow sign

Presentation:

1. "Sometimes in math we're not looking for an exact answer. We're looking for a number that is close to the exact answer, but not exact. We're going to do that kind of work today. What we'll be doing is called 'rounding' numbers. Today we are rounding to the nearest 10."

2. As before, form the square with the bead chain and compare it to the hundred square. Unfold it, and place the hundred square at the end of the chain. Have the child place the number arrows in their proper locations along the chain.

3. Select a number (73). Write that on the paper. Have the child locate it along the bead chain and leave the counter there.

4. Ask, "Which number arrow is nearest to 73?" Child should answer 70. Place the arrow sign above the 73 facing back toward 70. Say and write on the paper "73 rounds to 70."

5. Select another number (78). Repeat the procedure, but placing the arrow showing that 78 rounds up to 80.

6. After continuing with a few other numbers, choose 55. Ask, "Which arrow is nearest to 55?" A child should answer that it's exactly in the

middle or between the 50 and 60 arrows. Say, "When a number is exactly in the middle, we need a rule. The rule will be that we round this number up to the higher arrow, 60, which is the next 10. Place the arrow facing up toward 60 and write the answer on the paper. Continue with other numbers.

Extension: You can also round to the nearest ten with numbers to the hundred's place using the long 1000 chain (e.g., 732 rounds to 730).

Rounding to the Nearest Hundred*

Age: 7-9 years

Materials: long 10 chain (1000) and number arrows, 10 hundred squares, 1000 cube, counter, 2 arrow signs

First Presentation:

1. Do all of the steps for the regular presentation with the 1000 chain, folding the chain and showing the hundred squares, then stacking the 100 squares to compare to the 1000 cube.

2. Lay the chain out again and have the child place all of the arrows along the length of the chain.

3. Select a number (788) and ask, "Which ten is nearest to 788?" Child should answer 790. Review as much as needed. If the child understands this concept, have him/her remove all the blue arrows, leaving just the red 100 arrows and the green 1000 arrow.

4. Select a number (628). Ask, "Which hundred square is nearest to 628?" Child should place the counter at 628 and answer 600. Place the arrow sign above 628 facing down the chain toward the 600. Have the child say and write on the paper, "628 rounds to 600."

5. Continue with other numbers that will not be 50s.

Second Presentation:

1. Set up the 1000 chain as before. Select a number between 100 and 200 (123). Have the child place the counter at that bead.

2. Ask, "Which hundred is nearest to 123?" Child should answer 100. Place the arrow sign above 123 facing down the chain toward 100. Leave the arrow sign there.

3. Ask, "Which hundred is nearest to 177?" Child should place counter at 177 and answer 200. Place the arrow sign above the 177 pointing up the chain toward 200.

4. Say, "With 123 we rounded down to 100 and with 177 we rounded up to 200. Where along all of these numbers does rounding down change to rounding up?"

5. Explore with the child as needed to discover that 150 is exactly halfway between 100 and 200. Place the counter after the 150 bead. Recall the rule for numbers exactly in the middle (for tens, starting at 5 in the ones place you round up).

6. Move the arrow signs so that an arrow facing down the chain toward 100 is above 149 and an arrow sign facing up the chain toward 200 is over 150. If desired, make more arrows and have the child place them between other sets of hundreds.

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Graphing

Domain: Measurement and Data / Cluster: Represent and Interpret Data

First Grade Standard (1.MD.4)- Organize, represent and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than another.

Classifying the Geometric Cabinet in a Cluster Graph (or Venn Diagram)

Age: 6-7 years

Materials: Geometric Cabinet drawers (choose a variety of triangles, quadrilaterals and polygons), yarn, labels, pencil

Previous Learning: Geometric Cabinet drawers initial naming lessons

Presentation:

1. Before lesson begins create 3 large circles adjacent to one another, but not overlapping, on a large rug. Leave enough room at the top to place your row of triangles, quadrilaterals and polygons from the Geometric Cabinet. You could also do triangles, rectangles and squares if polygons have not been learned.

2. Ask children to name the shapes as you randomly place them in a row at the top of the rug. Review as needed (can just name triangle, not equilateral triangle, etc.).

3. Tell children that today you are going to sort the shapes into different groups or categories.

4. Ask the children which shapes have 3 sides? When they answer triangles, put the label for "triangles" above the first circle. Ask the children which shapes have 4 sides? When they answer quadrilaterals, put the label for "quadrilaterals" above the second circle. Ask the children which shapes have more than 4 sides? When they answer polygons (or name pentagon, etc.), put the label for "polygons" above the third circle.

5. Have the children take turns sorting the shapes into the correct categories.

When the sorting is complete do an analysis of the shapes. Ask, "How many triangles are there? How many quadrilaterals (or rectangles and

squares)? How many polygons? Which category has the most shapes? Which category has the least?
How many triangles and quadrilaterals are there?" You can also extend by comparing and asking how many more or less are in different categories.

Variations**:** To simplify, start with only two categories, rather than three. To extend, include shapes like curved figures and circles that will not fit under the headings and need to be placed outside of the circles.

Graphing

Classifying the Geometric Solids in a Cluster Graph (or Venn Diagram)

Domain: Measurement and Data / Cluster: Represent and Interpret Data

First Grade Standard (1.MD.4)- Organize, represent and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than another.

Age: 6-7 years

Materials: Geometric Solids, yarn, labels, pencil

Previous Learning: Geometric Solids

Presentation:
1. Before lesson begins create 2 large circles adjacent to one another, but not yet overlapping on a large rug. Have extra yarn around each circle so that you can create the overlapping area of the two circles later (Venn Diagram). Leave enough room at the top to place your row of Geometric Solids.

2. Ask children to name the Geometric Solids as you place them in a row at the top of the rug. Review names as needed.

3. Tell children that today we are going to sort the solids into different groups or categories based on whether they have straight or curved surfaces. Demonstrate that a cube has flat surfaces, while a sphere has a curved surface. Place the label for "flat surface" over one circle and the label for "curved surface" over the other circle.

4. Have the children take turns sorting the solids into the correct categories, starting with those that have only straight or curved surfaces, then introduce those with both.

5. Ask the children to analyze the cylinder and cone. They should note that they have flat and curved surfaces. Take the extra yarn from the adjacent sides of the circles, open them up and overlap to create Venn Diagram circles with the shared section in the center. Place the label for "both curved and flat surfaces" above this section and have the children put the cylinder and cone inside.

6. When the sorting is complete do an analysis of the solids. Ask, "How many solids with flat surfaces are there? How many solids with curved surfaces are there? How many solids with flat and curved surfaces are there? Which category has the most solids? Which category has the least? You can also extend by comparing and asking how many more or less are in different categories.

Graphing

Domain: Measurement and Data / Cluster: Represent and Interpret Data

Second Grade Standard (2.MD.10)- Organize, represent, and interpret data with up to four categories; complete picture graphs when single-unit scales are provided; complete bar graphs when single-unit scales are provided; solve simple put-together, take-apart, and compare problems in a graph.

Creating a Real Object Grammar Symbol Graph

Age: 7-8 years

Materials/Preparation: Different colors of construction paper (to match the Montessori grammar symbols) cut into 3-inch squares and arranged on a rug in horizontal or vertical rows of 10; paper or wooden grammar symbols of varying amounts from 2-10 (or a variety of other small items to sort and count on the graph); numeral cards (1-10);
blank paper bar graph (numbered 1-10); grammar symbol stencil; colored pencils

Presentation 1: Real Object Graph and Picture Graph
1. Explain that you are going to sort the grammar symbols into the squares you have arranged in rows and that each square will hold only one symbol of the same color.

2. Give each child the opportunity to sort a set of grammar symbols onto the correct colored squares. Initially, you may want to start with only 4 or 5 different symbols.

3. Once the symbols are sorted, remove any extra empty squares from each row and begin analyzing the graph. Ask the children, just visually first, which of the symbols do we have the most of and which the least?

4. Tell the children that they will now count exactly how many are in each row, but first we are going to number one side of our graph to show how many squares we have in all. Lay out the numeral cards from 1-10 along the horizontal or vertical axis (depending on how you have arranged this graph), making sure to place them evenly.

5. Ask each child to count how many symbols they placed in the squares and note the same number is to the side of the squares.

6. Use a blank bar graph (numbered 1-10 on the vertical axis for vertical graph) to demonstrate how you can create a picture graph for our object graph. Write the name of each symbol on the horizontal axis (noun, article, etc.). Use the stencil and colored pencils to draw as many symbols in your paper graph as are on the object graph for each.

7. Analyze the finished graph by asking questions. "How many do we have in the graph for the most symbols? How many for the least symbols? How many nouns and articles do we have together?" "How many more (or less) _____ are there than _____?"

8. The children could complete their own picture graph in the lesson or a version of the work can be put on the shelf for children to lie out and complete the picture graph independently

Variations: To simplify, stop before introducing the picture graph (first graders could typically do this). To extend, use more symbols or objects of different categories; add more analysis questions and extend with comparisons using inequality symbols (noun>article); demonstrate on a vertical and horizontal graph

Presentation 2: Real Object Graph and Bar Graph
This presentation proceeds as above with other objects, but introduces the concept of a bar graph with one square being colored in to represent each object, rather than drawing pictures.

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Graphing

Domain: Measurement and Data / Cluster: Represent and Interpret Data

Third Grade Standard (3.MD.3)- Create scaled picture graphs to represent a data set with several categories. Create scaled bar graphs to represent a data set with several categories. Solve two-step "how many more" and "how many less" problems using information presented in the scaled graphs.

[*Note: versions of this presentation can be used with upper elementary students using increasing complexity of operations and problems to meet the Fourth Grade Standard 4.MD.4.]*

Creating Scaled Picture and Bar Graphs

Age: 8-9 years (or older)

Materials/Preparation: blank bar graphs or graph paper, pencil

At this stage, the children are familiar with pictographs and bar graphs and are ready to move beyond real objects to the more abstract level of graphing data directly onto paper. Children at this age love to do surveys and this is a perfect opportunity for graphing data sets with several categories and with a scale that has intervals of 2, 5, 10, etc.

As an introduction, meet with the children to graph some data together, demonstrating a scale with intervals of 2 or 5. Demonstrate how to represent quantities by shading in squares on the bar graph or by creating a symbol key and drawing a picture graph. Also, demonstrate how to represent data with a picture graph, but emphasizing the need to clearly represent a half with their symbol. For example, if a smiley face ☺ represents 2 people on their graph, they will need to draw a half smiley face to represent odd numbers like 1, 3, 5, etc.

Children will also need practice answering analysis questions. Ask questions that require students to compare quantities and to solve one- and two-step "how many more" and "how many less" problems from the graph you complete together and the graphs they complete independently.

Guide the children in preparation for their own survey and graphs. For example, if they choose to survey the class of 28 children to find out how

many of each of several different kinds of pets each student has, they might only need a scale with intervals of 2 and each square will represent 2 pets as they graph their results. Discuss what to do when they have an uneven number in the result. If there are 5 students who have cats, fill in 2 and a half squares to represent 5.

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Division with the Pegboard

**** These lessons were shared with Xavier University's Montessori Teacher Education Program by Martha McDermott***

Domain: Operations and Algebraic Thinking/Cluster: Represent and solve problems using multiplication and division

Third Grade Standard (3.OA.2) - Interpret whole number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each.

Domain: Numbers and Operations in Base Ten/Cluster: Use place value understanding and properties of operations to perform multi-digit arithmetic with whole numbers less than or equal to 1,000,000.

Fourth Grade Standard (4.NBT.6) - Find whole number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

[Note: Presentation 4 applies to Fifth Grade Standard 5.NBT.6 - Find whole number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.]

Age: 7-10 years

Materials: Pegboard; box of green, blue and red pegs; cups in the same hierarchical colors; skittles in green, blue and red; paper and color-coded pencils for recording

Presentation 1: Short Division, First Passage

1. Relate pegs to the hierarchical material. This can be done in conversational manner.
2. Write an operation, e.g., 9764 ÷ 4. In this passage, write dividend, divisor, and quotient only.

3. Put the required number of pegs in each cup, matching hierarchical colors to each digit of the dividend. On the upper left hand rim of the pegboard, arrange four green skittles.
4. Take the thousands cup and place it above or on the pegboard. Begin sharing nine thousands among the four skittles, giving each two with one remaining. Write the quotient "2" next to the written problem, using a green colored pencil. (*Note: pegs already distributed can remain on the board.)*
5. Exchange the one remaining unshared thousand peg for 10 red hundred pegs, depositing them in the red cup along with the seven hundreds from the dividend. Turn the thousands cup upside down and remove it from the board.
6. Place the hundreds cup on the board and share out the 17 pegs among the four skittles. Write a "4" in the quotient (with red pencil) and continue same process of exchanging, sharing and recording with tens and units until problem is complete (9764 ÷ 4 = 2441).

Presentation 2: Short Division, Second Passage

This passage begins just as the first passage, using the problem 25357 ÷ 6. Since the initial two ten-thousand pegs cannot be shared out equally among six skittles, an exchange is made immediately at the outset of the problem. The number of pegs available to share out each time is recorded on the paper underneath the problem, as such:

25357 ÷ 6 = 4226
13
15
37
1 remainder

Presentation 3: Short Division, Third Passage

This passage is performed the same as the second passage, although the recording now includes the number of pegs used each time:

25357 ÷ 6 = 4226
24
13
12
15
12
37

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36
1

Presentation 4: Division with a Two-digit Divisor

Review the Golden Bead Division lesson when children wore blue and green ribbons for two-digit divisors. Relate this now to the skittles.

1. Record dividend and divisor for a problem, e.g., 37464 ÷ 24 =
2. Place pegs in cups for dividend and two blue and four green skittles on rim of pegboard
3. Place cups for ten-thousands and thousands on the board and share out ten-thousands to blue skittles and thousands to green skittles. Record the first digit of the quotient as 1 (in green for thousands – *if necessary, review that the answer in a division problem is based on the amount that one unit skittle receives*).
4. Exchange remaining ten-thousand peg for 10 thousands and remove and overturn empty ten-thousand cup. Bring down the hundreds cup and continue sharing, with highest value remaining pegs going to the blue skittles and the other to the green. With each pass, record quotient until problem completed: 37464 ÷ 24 = 1561.

Note: subsequent passages can be done if necessary to add in the gradual recording of pegs used and available as with one-digit divisors. Full notation would look like this:

37464 ÷ 24 = 1561
24
134
120
146
144
24
24

Presentation 5: Group Division

Review concept in conversation and with material, e.g., how many groups of 12 in 36, etc. Now, how many groups of 24 in 8847? Pegs are put in appropriate cups, skittles are set up. The divisor represents one group only. The problem is examined. How many groups of 2 (tens) in 8? The answer looks like 4. So 4 green pegs are given to each skittle. Now how many 4's in 8 (hundreds)? The answer is 2. But this does not balance with the blue skittles, so 2 green pegs are removed, one is exchanged for

hundreds. So we continue to find how many 4's in 18 (8 + 10). A balance is achieved at 3. This answer is recorded, 24 x 3 is calculated and subtracted from 88:

8847 ÷ 24 = 3
72
164

The 1 thousand is exchanged for hundreds. Now find how many 2's (20) in 16 hundreds. Looks like 8. Pegs are put out. How many 4's in 4, answer 1. Again an imbalance is present. Gradually pegs are removed from the blue skittles. Changes are made for tens and when a balance is created between tens and ones, then an answer is achieved. It is 6:

8847 ÷ 24 = 36
72
164
144
207

Two hundreds are exchanged for tens. Groups of 20 are created. Groups of 4 are created. Until a balance is achieved there is no final answer. This happens at 8. The remainder is 15:

8847 ÷ 24 = 368
72
164
144
207
192
15 remainder

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The Checkerboard Challenge Game

This work is an extension to the traditional Checkerboard multiplication lessons. It can be introduced once a student has reached a high level of proficiency with one and two-digit multipliers. While this would typically be introduced in the Montessori math curriculum in later Lower Elementary (ages 8-9), it addresses the Fourth Grade Standard 4.NBT.5 (Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.).

The teacher prepares colored graph paper cut into the geometric representations of the bead bars for various multiplication facts. For example, a rectangle that is four graph squares long by 3 squares wide would represent 4x3. The colored paper is green, red and blue to match the place value color pattern of the checkerboard.

For this game the child arranges the colored graph paper to create a multiplication problem following the place value color pattern for units (green), tens (blue), hundreds (red), thousands (green), etc. The paper must be arranged so that the multiplier is the same for each place value. The rectangles and squares are oriented next to each other in a row so that like sides match.

Once the child has arranged the paper to create the multiplication problem she can place the white and gray tiles along the Checkerboard to represent the multiplicand and multiplier and solve the equation.

Increase the challenge by adding a two or three-digit multiplier with a second row of graph paper matching the pattern of the second row of the Checkerboard, starting with the blue tens place above the green units place of your first row. For example, all of the first row might be 3 squares high to represent the multiplicand x3 for the units of the multiplier and all of the second row (starting with the blue tens place) are 4 squares high to represent x4 for the tens place of the multiplier. This represents the 2-digit multiplier x43.

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Extracting Square Root using the Stamp Game

Age: 8-9+

Materials: Stamp Game

Presentation:

1. Review the concept of a squared number (a number multiplied by itself). Introduce the idea that one can take a number and find out, or "extract," the number that, when multiplied by itself, results in that number. This is called a "square root."

2. Start with a three-digit number, e.g., 121 (*note: initial presentation and work should utilize perfect squares above 100. Avoid beginning with squares whose root includes a "0" digit, as these require placeholders in the pattern)*. Take out the stamps representing this number (one hundred stamp, two ten stamps, and 1 unit stamp).

3. Explain that by laying out the stamps in the shape of a square, we will be able to find the root of the square. Begin with the highest place value, the hundreds. Lay out the hundred stamps on a rug in a square pattern. If there are any extra hundreds beyond what is needed to form a square, these should be exchanged for 10 each of the ten stamps and added to the ten stamps already out.

4. Lay out the ten stamps below and to the right of the hundred stamps to form two rectangles. These should be built evenly, with the same number of ten stamps in each rectangle. If there are extra ten stamps beyond what can be built evenly, exchange them for 10 each of unit stamps and add these to the unit stamps already laid out.

5.) Lay out the unit stamps to fill in the square shape between the rectangles of ten stamps. Once the square shape is complete, the square root can be read across the base of the square. If any units remain after the square is complete, these are a remainder.